Vibration Control

Topics:
• Introduction to Vibration Control
• Methods of Vibration Control
• Vibration Isolation
• Rigidly Coupled Viscous Damper
• Elastically Coupled Viscous Damper
• Undamped Vibration Absorber
• Forced Damped Vibration Absorber
Introduction Vibration Control

- There are numerous Sources of Vibration in an Industrial Environment
- Presence of Vibration leads to
  - Excessive wear of bearings,
  - Formation of cracks,
  - Loosening of fasteners,
  - Structural and mechanical failures,
  - Frequent and costly maintenance of machines,
  - Electronic malfunctions
- Exposure of Humans leads to Pain, Discomfort and Reduced efficiency.

Hence it is necessary to eliminate or reduce vibration
Methods of Vibration Control

- Avoid Resonance
- Balancing / Control of Excitation Forces
- Adequate Damping
- Vibration Isolation
- Vibration Absorber
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Vibration Isolation

Isolating the structures from vibration is very important

- Accuracy of the machines
- Comfort levels of the passengers
- Transmission of vibrations to other nearby equipment
- Sound Generated due to the vibration is to be in limits
- Vibration of the buildings due to the equipment present in them
Vibration Isolation

Vibration Isolation with Rigidly Coupled Viscous Damper

\[ F = \sin \omega t \]

**Periodic Force**

\[ f_{tr} = KX \sin (\omega t - \phi) \rightarrow c\omega X \cos (\omega t - \phi) \]

\[ = F_{tr} \sin (\omega t - \phi - \beta) \]

\[ F_{tr} = \sqrt{(KX)^2 + (c\omega X)^2} \]

\[ = KX \sqrt{1 + (2\xi r)^2} \]

**Transmitted Force**

**Phase Angle**

\[ \beta = \tan^{-1} \left( \frac{c\omega}{k} \right) = \tan^{-1} (2\xi r) \]

**Phase Lag**

\[ \alpha = \tan^{-1} \left( \frac{2\xi r}{1 - r^2 + (2\xi r)^2} \right) \]

**Transmissibility**

\[ \frac{F_{tr}}{F} = \sqrt{\frac{1 + (2\xi r)^2}{(1 - r^2)^2 + (2\xi r)^2}} \]
Vibration Isolation

Vibration Isolation with Elastically Coupled Viscous Damper

Force Transmitted $\mathbf{f}_{tr} = Kx \leftrightarrow K_1x_1$

To Ground

$$\mathbf{X} = \frac{F}{K} \frac{\sqrt{1 + (2\xi_r\mu)^2}}{\sqrt{(1 - r_r^2)^2 + (2\xi_r(1 + \mu - r_r^2\mu))^2}} e^{-i\phi}$$

$$\mathbf{X}_1 = \frac{F}{K} \frac{2\xi_r\mu}{\sqrt{(1 - r_r^2)^2 + (2\xi_r(1 + \mu - r_r^2\mu))^2}} e^{-i\beta}$$

$$\mathbf{f}_{tr} = F \frac{\sqrt{1 + (2\xi_r\mu)^2} \cos(\omega t - \phi) + 2\xi_r \cos(\omega t - \beta)}{\sqrt{(1 - r_r^2)^2 + (2\xi_r(1 + \mu - r_r^2\mu))^2}}$$

$$\mathbf{f}_{tr} = F_{tr} \cos(\omega t - \psi)$$

Transmissibility

$$F_{tr} = F \frac{1 + 4(1 + \mu)^2 \xi_r^2 r_r^2}{\sqrt{(1 - r_r^2)^2 + 4\xi_r^2 r_r^2(1 + \mu - r_r^2\mu)^2}}$$

Phase Lag

$$\tan^{-1} \frac{2\xi r r_r^3}{(1 - r_r^2) + 4(1 + \mu) \xi_r^2 r_r^2 (1 + \mu - r_r^2\mu)}$$
Vibration Isolation

Vibration Isolation with Elastically Coupled Viscous Damper

A plot of transmissibility ratio \( F_r / F \) is shown for a value of \( \mu = 0.333 \) and values of \( \xi \) varying from zero to infinity.

\[
\frac{1}{r_c^2 - 1} = \frac{1}{1 - \frac{\mu}{\mu + 1} r_c^2}
\]

\[
r_c = \sqrt{\frac{2(1 + \mu)}{1 + 2\mu}}
\]

\[
TR_{\xi=0} = TR_{\xi=\infty}
\]

Optimum value of damping \( \xi_{\text{opt}} \) is obtained from the following condition

\[
\frac{d(TR)}{dr}_{r=r_c} = 0
\]

and is given by

\[
\xi_{\text{opt}} = \frac{\sqrt{2(1 + 2\mu)/\mu}}{4(1 + \mu)}
\]

The value of transmissibility with \( r = r_c \) and \( \xi = \xi_{\text{opt}} \) is the minimum value of the maximum possible transmissibility and is same as

\[
(TR)_{\text{minima}} = (1 + 2\mu)
\]
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The ratio of amplitudes is given by

Let $q = \sqrt{K_1/m_1}$, $X_{st} = F/K_1$, and $\mu = m_1/m$, mass ratio, then

$$X = \frac{F(K_1 - m_1\omega^2)}{(K + K_1 - m\omega^2)(K_1 - m_1\omega^2) - K_1^2}$$

$$X_1 = \frac{FK_1}{(K + K_1 - m\omega^2)(K_1 - m_1\omega^2) - K_1^2}$$

We note that $X=0$ at $\omega=\omega_0$

Design the system such that $\frac{k_1}{k} = \frac{m_1}{m} = \mu$

Then amplitude of vibration of absorber becomes

$$X_1 = -\frac{X_{st}}{\mu} = -\frac{F}{K_1}$$
It is to be observed that, the vibration of main mass becomes zero at the condition \( \frac{k_1}{k} = \frac{m_1}{m} = \mu \)

This means that the absorber system absorbs all the energy of the parent system.

Hence it is called “Dynamic Absorber”.

The frequency of the combined system is

\[
\left(1 + \mu - \frac{\omega_n^2}{p^2}\right) \left(1 - \frac{\omega_n^2}{p^2}\right) - \mu = 0
\]

And the 2 natural frequencies are

\[
\omega_{n1,2}^2 = p^2 \left[1 + \frac{\mu}{2} \pm \sqrt{\mu + \frac{\mu^2}{4}}\right].
\]
Natural Frequency variation of dynamically absorbed system
Frequency response of both the masses
Practical implementation of dynamic vibration absorber

- A beam attached with cantilevers with tunable masses

- Tuned absorber system, because the position of mass on the cantilever beam can be changed
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A forced damped absorber configuration is given below. The equations of motion are given by:

Defining the system properties.
Forced Damped Vibration Absorber

The solution of $X$ gives

$$\frac{\bar{X}}{X_{st}} = \frac{\mu \left( r_n^2 - r_f^2 \right) + i2\xi r_f}{\mu \left( 1 - r_f^2 \right) \left( r_n^2 - r_f^2 \right) - \mu^2 r_n^2 r_f^2 + i2\xi r_f \left( 1 - r_f^2 - \mu r_f^2 \right)}$$

We note that

$$\left| \frac{X + iY}{P + iQ} \right| = \sqrt{\frac{X^2 + Y^2}{P^2 + Q^2}}$$

Hence

$$\frac{X}{X_{st}} = \sqrt{\frac{\mu^2 \left( r_n^2 - r_f^2 \right)^2 + (2\xi r_f)^2}{\mu^2 \left[ \left( 1 - r_f^2 \right) \left( r_n^2 - r_f^2 \right) - \mu r_n^2 r_f^2 \right]^2 + (2\xi r_f)^2 \left( 1 - r_f^2 - \mu r_f^2 \right)^2}}$$

For damping value at 0 the equation reduces to previous undraped case and at infinity both masses got locked together and become rigid.
Solving the above equation, we get

\[ r_{f1,2}^2 = \frac{1}{1 + \mu} \left[ 1 \pm \frac{\mu}{2 + \mu} \right] \]

All Curves with different Damping pass through points P and Q

Hence it is possible to find the optimum Damping value
The optimum damping value is given by

\[
\xi^2 = \frac{3}{8} \left( \frac{\mu}{1 + \mu} \right)^3
\]

Which is obtained by differentiating equation with \( r_f \)

Thus the frequency response of a tuned absorber is given
A delicate instrument of mass $m=15\text{kg}$ is separated from its rigid container using a packing material that has a stiffness $K=0.5\text{MN/m}$ and damping factor $\xi$. During handling, the container gets dropped from a height of $h=5\text{m}$ and it comes to rest instantaneously as it hits the floor. Obtain expressions for maximum deflections of packing material and maximum acceleration of the instrument and compare these values for damping factor $\xi$ of 0.02 and 0.3. Clearance available limits the maximum packing material deflection to 0.2m and the maximum acceleration of the instrument is limited to 20g. Design an isolator to fulfill the above requirements.
Assignment

2 A rotating machine of mass 650 kg, operating at a constant speed of 1500 rpm, has an unbalance of 0.12 kglm. If the damping in the isolators is given by damping ratio of $\xi = 0.08$, determine stiffness of the isolators so that the transmissibility at the operating speed is less than or equal to 0.15. Also determine the magnitude of the force transmitted.

Consider an I.C. engine with a flywheel operating at 2000 rpm, the mass moment of inertia of the engine cylinders lumped together is 0.55 kglm$^2$. The flywheel inertia is relatively large, therefore it can be assumed to be grounded. The system is found to be in resonance with the fifth engine order excitation torque of 1 kNm amplitude. Design a dynamic torsional vibration absorber so that the resulting two natural frequencies of the system are at least 25 percent away from the excitation frequency. The maximum shear stress in the absorber shaft is not to exceed 20 MPa. Also determine the amplitude of the absorber mass.
Assignment

Figure shows a rotating machine mounted rigidly on a concrete block which in turn is mounted on four springs. The masses of the machine and the concrete block are 500 kg and 1200 kg respectively and the mass moments of inertia about their respective centres of gravity are 45 kgm$^2$ and 100 kgm$^2$. The positions of the two CG’s are shown. The excitation is due to an unbalance of 0.1 kgm in the rotor.

The operating speed of the machine is 1000 rpm. Assume the lateral stiffness of the springs to be equal to the stiffness in the vertical direction. The maximum permissible static deflection is 15 mm.

(a) For the purpose of vibration isolation it is desired that the minimum value of the frequency ratio $\omega/p$ is 2.5, where $p$ is largest natural frequency. What should be the value of $b$?

(b) Determine the steady state vibration amplitudes.

(c) What is the maximum force transmitted to the foundation, through each spring?
Acknowledgements

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